Reliability Estimation Including Redesign Following Future Test for an Integrated Thermal Protection System

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1. Abstract
Considering the effect of epistemic uncertainty on reliability at the design stage leads to a high probability of failure estimate. This probability is greatly reduced by post-design measures, such as redesign following tests. In this paper, we propose a methodology to accurately estimate the probability of failure at design stage by including the effects of future processes, such as tests and redesign. We expand a previous study applied to a single failure problem to multiple failure modes problem. An Integrated Thermal Protection System (ITPS) is one of concepts of the thermal protection systems for reusable vehicles, which is imposed to various failure modes simultaneously during atmospheric reentry. The proposed method is applied to an ITPS example, comparing with traditional deterministic approach. It is demonstrated that incorporating the effects of future redesign provides a significantly better accuracy of probability of failure estimate. Also, modeling probabilistic redesign brings a potential large mass reduction even after compensating for unconservative designs. These insights allow the designer to select a better initial design

2. Keywords: Uncertainty reduction, Post design test, Thermal protection system, Probability of failure

3. Introduction
Traditionally aerospace systems have been designed under deterministic approach, employing factors of safety to compensate for design and operational uncertainty. This approach does not provide the best tradeoffs of reliability and performance. There is a growing interest in replacing deterministic design approach with non-deterministic design approaches, which account for various sources of uncertainty. Then, reliability, i.e., probability of failure, may be calculated. The non-deterministic design approach, also known as Reliability-based Design Optimization (RBDO), allows quantitative tradeoffs between reliability and system performance, e.g. life-cycle cost [1].

Another important advantage of non-deterministic design approach is that one can trade off uncertainty reduction against design changes to deal with given uncertainty. Epistemic uncertainty, e.g., errors in analytical calculation which stem from lack of knowledge, can be reduced by additional knowledge from post-design processes, such as tests, inspections, and maintenance. Aleatory uncertainty, e.g., variability in material properties, can be reduced or truncated by quality control. Research on modeling the effect of the uncertainty reduction by post-design processes have been studied and showed the uncertainty can be substantially reduced through the processes (Refs. [2-5]). More recently, Acar, et al. [6] demonstrated that post design tests along with redesign can greatly reduce the probability of failure of the structure. Sankararaman, et al. [7] proposed an optimization problem of test resource allocation for multi-level and coupled systems.

Without modeling future uncertainty reduction, the probability of failure estimate at design stage is highly inaccurate and overly conservative. Matsumura et al. [8] showed that including the effect of all possible epistemic uncertainties embedded in a hierarchical complex system leads a substantially higher probability of failure estimate by orders of magnitude compared to the actual probability of failure. Since the reliability estimation is one of critical elements of tradeoff against the system performance, it is crucial to accurately estimate the probability of failure at initial design stage. However, current RBDO frameworks do not incorporate the effect of uncertainty reduction for reliability estimation at design stage.

Previously, Villanueva [9] proposed a method to incorporate the effect of a future single thermal test followed by insulation re-design on initial reliability estimates for an Integrated Thermal Protection System (ITPS). In this study, it was shown that the redesign following future test can reduce the probability of failure by orders of magnitude.

ITPS is also a multi-functional structure which protects a vehicle from aerodynamic heating during reentry and simultaneously works as a load bearing structure. Therefore, there is a coupling between thermal failure and structural failures. In this paper, we expand the previously proposed method to deal with tests and redesign associated with multiple failure modes. Two additional failure modes, stress and buckling, are considered as well as thermal failure. The additional structural tests related to stress and buckling failures are modeled. Then, the
reduction in epistemic uncertainty due to the tests is modeled. If too conservative or too unconservative test results are obtained, the structure may be redesigned. So redesign criteria are described. Finally, all the modeled processes are incorporated into a Monte Carlo Simulation framework.

A general description of ITPS is described in Section 4. In Section 5, a calculation method of probability of failure is explained. In this paper, we have two types of probabilities of failure, which are the designer’s estimated probability of failure and true probability of failure. Post design processes are showed in Section 6 comparing the probabilistic approach to the traditional deterministic approach. The procedures of modeling future processes are described in Section 7. In Section 8, an example problem is demonstrated and the results are discussed.

4. Integrated Thermal Protection System (ITPS)
Thermal protection systems of space vehicles protect a vehicle and payloads from extreme temperatures during atmospheric reentry. Integrated thermal protection system (ITPS) is a concept for reusable vehicles, which can provide structural load bearing function and insulation function simultaneously, intended to save weight. Modeling and optimization problems on ITPS have been studied (e.g. [10-13]).

The present ITPS concept is comprised of the top face sheet, the bottom face sheet, and the corrugate core filled with insulation material (Fig 1). In this paper, thermal analysis of the ITPS is done by 1-D heat transfer analysis, and the stress and buckling models are obtained from a past study [13]. The mass per unit area m, used for tradeoff against reliability, is calculated by Eq.(1), where \( \rho_T, \rho_B, \) and \( \rho_W \) are the densities of materials used for the top and bottom face sheets, and web, respectively. \( t_T \) is the thickness of the top face sheet and \( \theta \) is the corrugation angle.

\[
m = \rho_T t_T + \rho_B t_B + \rho_W t_W d_s \rho \sin \theta
\]

For redesign, the thicknesses of web \( t_W \), bottom face sheet \( t_B \) and foam \( d_s \), are selected as design variables.

We assume that tests of the structure will be conducted in order to verify the design. Observed data from the tests will be utilized to calibrate errors in analytical calculations related to the structural behaviors. Three post design tests are hypothesized here. First, a small ITPS panel will be tested in a vacuum chamber with a heat applied to the top face sheet. Then, the temperature of the bottom face sheet \( T \) will be measured to see if the temperature does not exceed the material allowable. Second, a thermal buckling test is conducted by applying heat to the top face sheet to measure the temperature difference between the top and bottom face sheets \( \Delta T \) that will cause buckling. Finally, mechanical and thermal stress test takes place to observe failure stress \( \sigma_L \) of test specimens under the replicated flight condition. This stress test is assumed to be repeated for three nominally identical panels.

![Figure 1: Integrated Thermal Protection Systems (ITPS)](image)

5. Probability of Failure Calculation
Probability of failure in flight can be obtained considering variations in the structural behaviors (Response: \( R \)) and the critical values of the response that cause failure (Capacity: \( C \), such as strength for the stress response). Actual probability of failure in flight, which is unknown to the designer, is calculated by formulations of limit states in Eq.(2). In this paper we call it true probability of failure in order to distinguish from designer’s estimated probability of failure.

\[
g_T^{\text{true}} = (1 + v_T)T_s^{\text{true-mean}} - (1 + v_T)T_r^{\text{true-mean}}
\]

\[
g_\sigma^{\text{true}} = (1 + v_\sigma)\sigma_s^{\text{true-mean}} - (1 + v_\sigma)\sigma_r^{\text{true-mean}}
\]

\[
g_\theta^{\text{true}} = (1 + v_\theta)\Delta T_s^{\text{true-mean}} - (1 + v_\theta)\Delta T_r^{\text{true-mean}}
\]

where \( v_T, v_\sigma, \) and \( v_\theta \) are variability in strengths due to material properties, \( T_s^{\text{true-mean}}, \sigma_s^{\text{true-mean}} \) and \( \Delta T_s^{\text{true-mean}} \) are the true mean strengths, \( v_T, v_\sigma, \) and \( v_\theta \) are variability in responses due to flight conditions, material properties and geometry of the structure. \( T_r^{\text{true-mean}}, \sigma_r^{\text{true-mean}} \) and \( \Delta T_r^{\text{true-mean}} \) are the true mean responses for a given structure.

Since the designer does not know those true values, he needs to estimate the probability of failure based on his calculations (denoted with subscript of “cal”). On top of that, due to the fact that analytical models have errors, the probability of failure should take into account the uncertainty of analytical errors (denoted as \( e_{\text{cal}} \)). Thus, the
limit state functions are reformulated as in Eq.(3). We call these as designer’s estimated probability of failure.

\[
\begin{align*}
g_{T\text{-est}} &= C - R = (1 + v_T)(1 + e_{T,\text{cal}})T_{cal\text{-mean}} - (1 + v_T)(1 - e_{T,\text{cal}})T_{cal\text{-mean}} \\
g_{S\text{-est}} &= C - R = (1 + v_S)(1 + e_{S,\text{cal}})S_{cal\text{-mean}} - (1 + v_S)(1 - e_{S,\text{cal}})S_{cal\text{-mean}} \\
g_{B\text{-est}} &= C - R = (1 + v_B)(1 + e_{B,\text{cal}})B_{cal\text{-mean}} - (1 + v_B)(1 - e_{B,\text{cal}})B_{cal\text{-mean}}
\end{align*}
\]

For the sake of simplicity, a scaled buckling criterion which is derived from a reference design in Ref [13] where 3-D finite element analysis was solved is used. Details of the criterion are described in Appendix. Note that all the error and variability components in Eq. (2) and (3) are random variables which follow related probability density function (PDF) distributions. We use Separable Monte Carlo simulation [14] for implementation.

### 6. Post Design Processes

Figure 2 compares deterministic and probabilistic post-design processes. In the deterministic approach, structures are designed based on design margin, i.e., safety factor. If the test indicates too conservative or too unconservative design, redesign takes place so as to reinstate the initial design margin. On the other hand, in the non-deterministic approach, designers model uncertainties and, calculate the probability of failure. Test data is utilized to calibrate the error distributions by Bayesian inference technique, and the probability of failure is updated resulting in a more accurate probability of failure estimate. The updated error distributions are also used for redesign under a constraint of retaining the initial probability of failure. For the purpose of comparison to see how we benefit from probabilistic approach, we also examine results of the deterministic approach from probabilistic point of view.

![Traditional Deterministic Approach vs. Non-Deterministic Approach](image)

**Figure 2: Comparison of post-design processes**

### 7. Simulating Future Processes at the Design Stage

A key concept of the proposed method is to simulate all possible realizations of a design candidate. In following subsections procedures of simulating future processes are described.

#### 7.1 Modeling Uncertainty and Tests

At the design stage, all that the designer knows about his design candidate is predicted structural responses from his analytical models. Therefore, we have to explore possible realizations based on the predictions. However, if we could model all possible uncertainties throughout the future processes, we can simulate the possible realizations of the design candidate. First, we model the related uncertainties, such as errors in the analytical models, variability of the structure, and errors in test observation.

**Step-1. Modeling errors in analytical models and variability in structure**

(1-1) Assume PDF distributions of true errors in analytical models
(1-2) Assume PDF distributions of variability in material properties and geometry due to manufacturing
(1-3) Assume PDF distributions of errors in measurement in the tests or test configurations

Next, we virtually simulate what happens in tests by using the above defined uncertainties. It is assumed that we can accurately measure the material properties and geometry of a given test specimen so that we can calculate the expected structural responses of the specimen.
Step-2. Constructing a virtual test specimen and simulate test observation
(2-1) Generate a specimen (geometry and material properties) accounting for randomness of variability
(2-2) Calculate structural responses of the specimen (temperature, stress, and buckling) by analytical models
(2-3) Calculate true structural responses accounting for randomness of errors in analytical models
(2-4) Calculate observed structural responses accounting for randomness of errors in measurement

Finally, all possible future realizations are explored by repeating the process.
Step-3. Explore all possible realizations
(3-1) Repeat Step-2 for finite times (e.g., 1000 times)

For example, in the heat transfer test we observe the temperature of the bottom face sheet. The true temperature of a given specimen, $T_{true}$, can be virtually calculated by the predicted value $T_{cal}$ as (Step 2-2 and 2-3)

$$T_{true} = (1 - e_{T,cal})T_{cal}$$

where $e_{T,cal}$ is one of the realizations of the true error in the analytical model. Since it is assumed that we can accurately measure the geometry and material properties of the specimen, there is no place where the variability in material properties and geometry works at this point. Note that negative error represents unconservative error in this paper. Similarly, the observed temperature, $T_{obs}$, can be produced with a random true measurement error, $e_{T,meas}$, as (Step 2-4)

$$T_{obs} = T_{true}/(1 + e_{T,meas})$$

7.2 Modeling Uncertainty Reduction due to Tests.
Bayesian inference is a statistical method in which observed evidence is used to estimate the degree of confidence in a hypothesis. For the thermal test of ITPS, Bayes formula provides the probability of a possible true temperature given the observed temperature from

$$P(T_{true}|T_{obs}) = \frac{P(T_{obs}|T_{true})P(T_{true})}{P(T_{obs})}$$

where the conditional probability in the numerator is also called the likelihood function. In terms of probability density function, the updated PDF of the true temperature, $f_{upd}(T)$, can be written as

$$f_{upd}(T) = \frac{l(T)f^{ini}(T)}{\int_{-\infty}^{\infty} l(T)f^{ini}(T) dt}$$

where $f^{ini}(T)$ is the initial probability density based on the calculation and the calculation error model. $l(T)$ is the likelihood function, the conditional probability density of seeing the test result $T_{obs}$ for the temperature $T$. Once the PDF of the true temperature is updated by the test data, the PDF of the computational error, $e_{T,cal}$, can be updated by replacing $T$ with $e_{T,cal}$ by modified Eq.(4)

$$e_{T,cal} = 1 - \frac{T}{T_{cal}}$$

The updated PDF of the error, then, is used for re-calculating the probability of failure and for redesign as well. In the same manner, the errors in the stress strength calculation, $e_{\sigma,cal}$, and the buckling calculation, $e_{\Delta T,cal}$, are also updated. For more details and illustrative examples of Bayesian inference for post-design test, the reader is referred to Ref [9].

7.3 Modeling Redesign
If too conservative or too unconservative test results are obtained, the structure may be redesigned. In deterministic approach, redesign is conducted based on safety factor, while in probabilistic design approach redesign is implemented with the statistical knowledge, i.e., probability of failure.

Deterministic Redesign
The designer determines the necessity of redesign based on discrepancy between his prediction and observed data. If the observed test results are out of acceptable bounds, redesign is needed. We assume that if two of the acceptance criteria in Eq. (9) are violated, the design is sent to redesign.

$$T_{low} \leq T_{obs} \leq T_{up} \text{ (Thermal test)}$$
$$\sigma_{low} \leq \sigma_{obs} \leq \sigma_{up} \text{ (Stress test)}$$
$$\Delta T_{low} \leq \Delta T_{obs} \leq \Delta T_{up} \text{ (Buckling test)}$$

4
For redesign, the analytical calculation is calibrated by a correction factor, e.g., \( T_{\text{obs}} / T_{\text{cal}} \). Then, a new design point is found by solving a safety factor based optimization problem. The objective function is to minimize the mass of the ITPS given in Eq.(1) remaining the initial factors of safety.

\[
\begin{align*}
\min_x \ m \\
s.t. \ SF_T(x) \leq SF_{T_{\text{init}}}, \ SF_{\sigma}(x) \leq SF_{\sigma_{\text{init}}}, \text{ and } SF_B(x) \leq SF_{B_{\text{init}}}
\end{align*}
\]

where \( x \) is the vector of the design variables, such as the thicknesses of web \( (t_w) \), bottom face sheet \( (t_b) \) and foam \( (d_g) \). \( SF_{\text{init}} \) is safety factor related to each failure mode.

Probabilistic Redesign

Redesign criterion is defined by bounds of acceptable probability of failure. If the updated probability of failure after uncertainty reduction, \( PF_{\text{est,up}} \), is out of the bounds, the structure needs to be redesigned.

\[
PF_{\text{low}} \leq PF_{\text{est,up}} \leq PF_{\text{up}}
\]

Unlike the deterministic redesign, instead of calibrating the analytical model, the updated error distributions are used for calculating the probability of failure. Redesign is performed by solving the following optimization problem. The constraint is to remain the initial probability of failure, \( PF_{\text{est,ini}} \).

\[
\begin{align*}
\min_x \ m \\
s.t. \ PF_{\text{est,up}}(x) \leq PF_{\text{est,ini}}
\end{align*}
\]

8. Example Problem and Results

8.1 Design Candidate

The ITPS is assumed to consist of the top face sheet and webs made of titanium alloy (Ti-6Al-4V), and the bottom face sheet made of beryllium. Saffil® foam is used as insulation between the webs. Table 1 shows the geometry of a design candidate for an example problem. Table 2 is a list of distributions of errors. Table.3 is safety factors and designer's estimated probabilities of failure of the design candidate. In this example, we assume that the designer examines 1000 possible realizations of the epistemic uncertainties, meaning that 1000 sets of random true errors are generated.

### Table 1: Geometry of design candidate

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web thickness</td>
<td>( t_w )</td>
<td>1.65 [mm]</td>
</tr>
<tr>
<td>Bottom face sheet thickness</td>
<td>( t_b )</td>
<td>6.90 [mm]</td>
</tr>
<tr>
<td>Foam thickness</td>
<td>( d_g )</td>
<td>72.88 [mm]</td>
</tr>
<tr>
<td>Mass</td>
<td>( m )</td>
<td>33.92 [kg]</td>
</tr>
</tbody>
</table>

### Table 2: Error setting

<table>
<thead>
<tr>
<th>Failure mode</th>
<th>Description</th>
<th>Symbol</th>
<th>Variability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error in calculation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thermal</td>
<td>Temperature calculation</td>
<td>( e_{T_{\text{cal}}} )</td>
<td>Uniform ±5%</td>
</tr>
<tr>
<td>Stress</td>
<td>Stress calculation</td>
<td>( e_{\sigma_{\text{cal}}} )</td>
<td>Uniform ±5%</td>
</tr>
<tr>
<td>Stress strength</td>
<td>Stress strength calculation</td>
<td>( e_{\sigma_{\text{strength}}} )</td>
<td>Uniform ±5%</td>
</tr>
<tr>
<td>Buckling</td>
<td>Temperature difference calculation</td>
<td>( e_{\Delta T_{\text{cal}}} )</td>
<td>Uniform ±5%</td>
</tr>
<tr>
<td>Error in test</td>
<td>Measurement and test configuration</td>
<td>( e_{T_{\text{meas}}} )</td>
<td>Uniform ±3%</td>
</tr>
<tr>
<td>Thermal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stress</td>
<td>Measurement and test configuration</td>
<td>( e_{\sigma_{\text{meas}}} )</td>
<td>Uniform ±3%</td>
</tr>
<tr>
<td>Buckling</td>
<td>Measurement and test configuration</td>
<td>( e_{d_{g,\text{meas}}} )</td>
<td>Uniform ±3%</td>
</tr>
</tbody>
</table>

### Table 3: Safety factors and probabilities of failure of design candidate

<table>
<thead>
<tr>
<th>Failure mode</th>
<th>Safety factor</th>
<th>Designer's estimated probability of failure ( ^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal</td>
<td>1.35</td>
<td>2.8x10^{-7} (1.14)</td>
</tr>
<tr>
<td>Stress</td>
<td>1.28</td>
<td>1.2x10^{-7} (0.33)</td>
</tr>
<tr>
<td>Buckling</td>
<td>1.42</td>
<td>3.3x10^{-7} (0.41)</td>
</tr>
<tr>
<td>System</td>
<td>-</td>
<td>1.6x10^{-7} (0.31)</td>
</tr>
</tbody>
</table>

\( ^* \) Number in parentheses is coefficient of variation calculated from 1000 repetitions.
Before incorporating the effects of future uncertainty reduction and redesign, the designer can examine 1000 possible true probabilities of failure using the sets of true errors. Figure 3 shows a histogram of 1000 possible true system probabilities of failure of the design candidate. It can be seen that 60% are below the designer’s estimated probability of failure, but 40% are above. Also, it is important to note that the maximum and minimum possible true probabilities of failure are underestimated compared to theoretical ones. This is due to the limitation of the number of samples.

8.2 Deterministic Approach
We assume that if more than two of the test results are out of the acceptable bounds (±2.5%) in Eq.(9), redesign is needed. As a result, 117 possible realizations are sent to redesign because of too unconservative test observations, while 148 are too conservative. As seen in Fig 4.(a) and (b), it is obvious that selections for redesign are not quite related to the true probabilities of failure, since no statistical information is available in deterministic approach. As seen in the result of redesign shown in Fig 4.(c), the mean of mass is increased by 0.4% (33.92 → 34.06 kg) to compensate the too unconservative designs. Also deterministic redesign reduces the variation in possible realizations of true probability of failure by 13% (coefficient of variation 0.72→0.63).

8.3 Probabilistic Approach
First we examine the effect of the uncertainty reduction after the tests. Figure 5 shows a histogram of the discrepancies between the estimated and true probabilities of failure before and after the uncertainty reduction. It is obvious that the magnitude of the discrepancies is significantly reduced after the tests. The standard deviation is reduced by magnitude factor of two (1.2×10^{-3} → 5.5×10^{-4}). Also, the maximum unconservative discrepancy is improved from 225% to 52%. The reason that the discrepancy after uncertainty reduction is biased is due to the assumption that the errors in test measurement are proportional to the values of observed data. This also makes the likelihood functions in Bayesian formula (in Eq.(7)) vary with respect to the values of observed data, resulting in
Probabilistic redesign also takes advantage of the uncertainty reduction. For the purpose of comparison to the deterministic redesign approach, redesign criteria are selected so that the same number of possible realizations is sent to redesign. The upper bound in Eq.(11) is selected as 2.1 times $PF_{est,up}$, and the lower bound is 0.74 times $PF_{est,up}$. Figure 6 shows the of redesign selections. Even though the redesign items are determined based on the designer’s estimated probability of failure, the selections also look reasonable on the histogram of true probability of failure. Figure 7 shows the histogram of 1000 realizations of true probability of failure after redesign. Comparing to the initially available information (Fig. 3), the variation is reduced by 27% (coefficient of variation 0.72 → 0.60). Moreover, as previously mentioned in Fig 5, these true probabilities of failure will be more accurately predicted after the tests. A great contribution of the probabilistic redesign is mass reduction. Even though compensating for the unconservative designs (intuitively additional mass is needed to make it safer), the mean of masses is still reduced by 5% (33.92 → 32.15 kg). This is significant compared to the safety factor based redesign where the mean of masses are increased after the redesign (Fig 4(c)).

From risk allocation point of view, Table 4 shows a comparison of probabilities of failure for each failure mode before and after redesign. It can be seen that the probability of failure for stress is increased after the deterministic redesign even though the stress failure is initially dominant. This is due to the fact that the deterministic redesign criteria are based only on design margin. It does not care which failure mode is dominant. On the other hand, it is observed that the probabilistic redesign offers a better risk allocation.

Even though these gains are obtained from the virtual simulation of the future processes, this knowledge allows the designer to find a better initial design point. He can select the initial design point not based on his initial predictions which include errors but on possible final realization of the design candidates after tests and redesign.
Figure 6: Redesign selection by probabilistic criteria

Figure 7: Histogram of possible true system probabilities of failure (PF) after redesign

Table 4: Risk allocation

<table>
<thead>
<tr>
<th></th>
<th>Thermal</th>
<th>Stress</th>
<th>Buckling</th>
<th>System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>$2.8 \times 10^{-3}$</td>
<td>$1.2 \times 10^{-3}$</td>
<td>$3.3 \times 10^{-4}$</td>
<td>$1.6 \times 10^{-7}$</td>
</tr>
<tr>
<td>Deterministic approach</td>
<td>$2.4 \times 10^{-4}$</td>
<td>$1.3 \times 10^{-3}$</td>
<td>$3.3 \times 10^{-4}$</td>
<td>$1.7 \times 10^{-1}$</td>
</tr>
<tr>
<td>Probabilistic approach</td>
<td>$2.4 \times 10^{-4}$</td>
<td>$8.8 \times 10^{-4}$</td>
<td>$3.3 \times 10^{-4}$</td>
<td>$1.4 \times 10^{-1}$</td>
</tr>
</tbody>
</table>

9. Conclusion
This study presented a methodology to include the effects of multiple future tests followed by redesign in order to accurately estimate probability of failure at design stage. In addition to the previous study, two additional structural tests are modeled and incorporated into uncertainty reduction and redesign optimization problems. With the example problem, first, it is demonstrated that modeling the uncertainty reduction by future tests provides a substantially better accuracy of probability of failure estimate. Second, probabilistic redesign gives not only a
smaller variation in realizations of possible true probability of failure (27% reduction in coefficient of variation) but also significant mass reductions. The proposed methods offer great insights on the future processes which enable designers to select a better initial design point based on future realization of the structure.

10. Acknowledgements
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11. Appendix: Buckling Criterion
Instead of constructing a fine simulation model for buckling prediction, we deploy a simplified scaling buckling criterion by utilizing a reference design which was optimized for similar constraints and with 3D finite element analysis [13]. For deriving the scaling buckling criterion, we assume that web is the weakest link because thermal consideration pushes it to be thin and that its buckling is overall Euler buckling.

If operational temperature difference between the top face sheet and the bottom face sheet, which is denoted as $\Delta T_{op}$, is greater than the one causes buckling, $\Delta T_b$, the structure fails. Therefore, a limit state of buckling can be written as

$$\frac{\Delta T_b}{\Delta T_{op}} < 1$$  \hspace{1cm} (A1)

By using data of the reference design, the left hand side of Eq.(A1) can be rewritten as

$$\frac{\Delta T_b}{\Delta T_{op}} = \frac{\Delta T_{b,ref}}{\Delta T_{op,ref}} \left( \frac{\Delta T_b}{\Delta T_{b,ref}} \right) \left( \frac{\Delta T_{op,ref}}{\Delta T_{op}} \right)$$  \hspace{1cm} (A2)

Since the reference design is designed to have a safety factor of 1.2 in terms of $\Delta T$, the first term in the right hand side of Eq.(A2) equals 1.2. In the last term of Eq.(A2), $\Delta T_{op,ref}$ is given from the reference design and $\Delta T_{op}$ is calculated by 1-D thermal analysis for a given structure. Assuming that buckling of ITPS is mainly occurred due to thermal compression load, the second term can be approximately equal to the ratio of Euler buckling load, $P_b$, to thermal compression load, $P_t$, as shown in Eq.(A3).

$$\frac{\Delta T_b}{\Delta T_{op}} \approx \frac{P_b}{P_t} \rightarrow \Delta T_b \approx \frac{P_b}{P_t} \Delta T_{op}$$  \hspace{1cm} (A3)

By substituting $P_b$ and $P_t$ into Eq.(A3)

$$\Delta T_b \approx \frac{P_b}{P_t} \Delta T_{op} = \frac{\left( \pi^2 EI \right)}{kE \Delta T_{op,ref}} = \frac{\left( \frac{\pi^2}{12} \cdot \frac{1}{\alpha} \cdot \frac{t_w^3}{(Kd_s)^2} \right)}{k \alpha = \frac{\pi^2}{12} \frac{t_w^2}{d_s^2}} = c_1 \left( \frac{1}{\alpha} \frac{t_w^2}{d_s^2} \right)$$  \hspace{1cm} (A4)

where $c_1 = \frac{\pi^2}{12kk}$. Finally, by substituting all components into Eq.(A1), the buckling criteria can be expressed as a function of material property and geometry of structure, and $(\Delta T)_{op}$.

$$1.2 \left[ \frac{1}{\alpha} \frac{t_w^2}{d_s^2} \right] \left( \frac{\Delta T_{op,ref}}{\Delta T_{op}} \right) < 1$$  \hspace{1cm} (A5)

In the limit state function formulation, Eq.(A5) is rewritten as

$$g = C - R = 1.2 \left[ \frac{1}{\alpha} \frac{t_w^2}{d_s^2} \right]_{ref} - \frac{\Delta T_{op,ref}}{\Delta T_{op}}$$  \hspace{1cm} (A5)

12. References


